

# Research Notes For Passion Project 2014

Mehmet

## Quantum Teleportation

From: Mehmet Yilmaz

PLEASE READ: (About Research notes for Passion Project 2014)

Main resources used to get research notes:

92% of this research was made from the two novels:

Dance of the photons from Einstein to quantum teleportation, By Anton Zeilinger  
The physics of quantum information, By David Bouwmeester, Artur Ekert, and Anton Zeilinger

8% of the research was from the Internet and mainly from Wikipedia

A lot of the research may be copied from the resources listed above, as remember that this research and notes were not 100% made by Mehmet Hadi Yilmaz. They were mainly inspired or rewritten from the resources listed above.

What is Quantum teleportation?

It is a process by which quantum information (the exact state of an atom or Photon) can be transmitted exactly from one location to another.

The scheme for quantum dense coding (made by Bennett and Wiesner) utilises entanglement between two qubits, where each individually has two orthogonal states,  $|0\rangle$  and  $|1\rangle$ . Note there are four possible Polarisation combination (or outcomes) for a pair of entangled particles;  $00, 01, 10$ , and  $11$ .

Identifying each combination with different info shows that we can encode two bits of info by manipulating both particles.

Natural units

Quantum Mechanics allows use to encode info in superpositions of the classical combinations (entangled Potentials), this is called entangled state and a convenient basis in which to represent the states for two particles in labeled 1 and 2, that is formed from the entangled Bell state.

By manipulating 1 (of the 2) particles we can encode two bits of info, this is done by doing the following:

$$|\Psi^+\rangle_{12} = (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2)/\sqrt{2}$$

$$|\Psi^-\rangle_{12} = (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2)/\sqrt{2}$$

$$|\Phi^+\rangle_{12} = (|0\rangle_1 |0\rangle_2 + |1\rangle_1 |1\rangle_2)/\sqrt{2}$$

$$|\Phi^-\rangle_{12} = (|0\rangle_1 |0\rangle_2 - |1\rangle_1 |1\rangle_2)/\sqrt{2}$$

- Bell State: Represents the most simple examples of entanglement
- A bit in Quantum Physics is the basic unit of info. A quantum bit can use 1, 0, and both 1 and 0 in binary code. Unlike a computer bit which can only use 1 and 0 in the binary code. (this is called a qubit).
- Also note 500 qubit holds more numbers than all the atoms in the universe we know now.

Alice and Bob each get one particle that are entangled, say in the state  $|\Psi^+\rangle_{12}$ . So if Bob were to transfer his particle (Particle 2) to Alice, then Bob would have to perform one out of four possible unitary transformations on his particle 2 alone.

The four transformations:

1. Identity operation (not changing the original two-particle state  $|\Psi^+\rangle_{12}$ )

2. State exchange ( $|0\rangle_2 \rightarrow |1\rangle_2$  and  $|1\rangle_2 \rightarrow |0\rangle_2$  and changing the two particle states to  $|\Phi^+\rangle_{12}$ )

3. State-dependent Phase Shift (differing by  $\pi$  for  $|0\rangle_2$  and  $|1\rangle_2$ ) and transforming to  $|\Psi^-\rangle_{12}$ )

4. State Exchange and Phase Shift together (giving the state  $|\Phi^-\rangle_{12}$ ).

(information found on Page 50 in the book, "The Physics of Quantum Information")

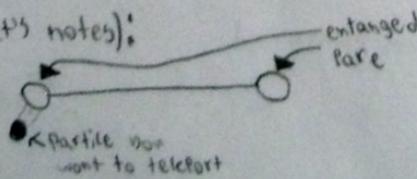
This shows that these four manipulations result in four orthogonal Bell states, distinguishable messages with 2-bit info can be sent.

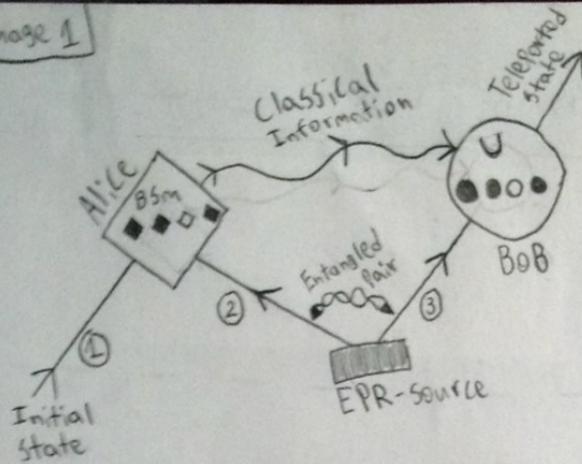
Image 1.1, it's known as the four maximally entangled two-qubit states or Bell state

There are 4 outcomes because of the following (Mehmet's notes):

- white white
- black black
- black white
- white black

This outcome happens when you do the following:





**Principle of Quantum teleportation:** Alice has a quantum system, Particle 1, in an initial state which she wants to teleport to Bob. Alice and Bob also share an ancillary entangled pair of particle 2 and 3 emitted by an Einstein-Podolsky-Rosen (EPR) source. Alice then performs a joint Bell state measurement (BSm) on the initial particle and one of the ancillaries, projecting them also onto an entangled state. After she has sent the result of her measurement as classical information to Bob, he can perform a unitary transformation (U) on the other ancillary particle resulting in it being in the state of the original particle. In the case of

quantum teleportation of a qubit, Alice makes a projection measurement onto four orthogonal entangled states (or the Bell state) that form a complete basis. Sending the outcome of her measurement, i.e. two bits of classical information, to Bob will enable Bob to reconstruct the initial qubit.

Alice wants to transfer particle 1 to Bob, but suppose she cannot deliver the particle directly to him. Note that any quantum measurement performed by Alice on her particle will destroy the particle's quantum state, which will not allow Bob to get the particle's info. So what does Alice do? Well let's say particle 1 has a quantum state, with the qubit  $|I\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$ , where  $|0\rangle$  and  $|1\rangle$  represent two orthogonal states. Particles 2 and 3 (EPR pair), were particle 2 is given to Alice and particle 3 is given to Bob.

$$|I\rangle_{23} = \frac{1}{\sqrt{2}}(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3)$$

Note that if one of these particles were to be measured, well in an entangled state, the particle projects it onto a certain state, which can be any normalised linear superposition of  $|0\rangle$  and  $|1\rangle$ , then the other has to be in the orthogonal state. The equation above:

- On the far right hand side the phase difference is  $\pi$ , which results in the minus sign.

- This shows that the statement of orthogonality is independent of the basis chosen for the polarization measurement.

Although initially particles 1 and 2 are not entangled, their polarization state can be expressed as a superposition of four maximally entangled Bell states, these states form a complete orthogonal basis. The total state of 3 particles written as:

$$\begin{aligned} |I\rangle_{123} &= |I\rangle_1 \otimes |I\rangle_{23} = \frac{1}{2} [ |I\rangle_{12} (-\alpha|0\rangle_3 - \beta|1\rangle_3) + \\ &\quad |I^+\rangle_{12} (-\alpha|0\rangle_3 + \beta|1\rangle_3) + \\ &\quad |I^-\rangle_{12} (\alpha|1\rangle_3 + \beta|0\rangle_3) + \\ &\quad |I^+\rangle_{12} (\alpha|1\rangle_3 - \beta|0\rangle_3) ] \end{aligned}$$

Alice then performs a Bell state measurement (BSm) on particles 1 and 2, she then projects her two particles onto one of the four Bell states. Bob's particle's measurement will be directly related to the initial state. For example if Alice's Bell state measurement is  $|I^-\rangle_{12}$ , then particle 3 in the hands of Bob would be in the state  $\alpha|1\rangle_3 + \beta|0\rangle_3$ . Now Alice has to inform Bob via a classical communication channel on her measurement result and Bob can perform the appropriate unitary transformation (U) on particle 3 in order to obtain the initial state of particle 1. This completes the teleportation protocol.

Note that during the teleportation procedure, the values of  $\alpha$  and  $\beta$  are unknown. By doing the Bell state measurement Alice does not get any info about the teleported state. By doing the Bell state measurement, you would only get a transfer of the quantum state. Also note when the Bell state measurement is done on Particle 1, it then loses its initial quantum state because it becomes entangled with Particle 2. This makes the state  $|I\rangle_1$  be destroyed. This obeys the no-cloning theorem, when the measurement is taken place, then Particle 1's state will be unknown to anyone. In this case Particle 1 itself is in an entangled pair and there are no well-defined properties.

#### Other Notes:

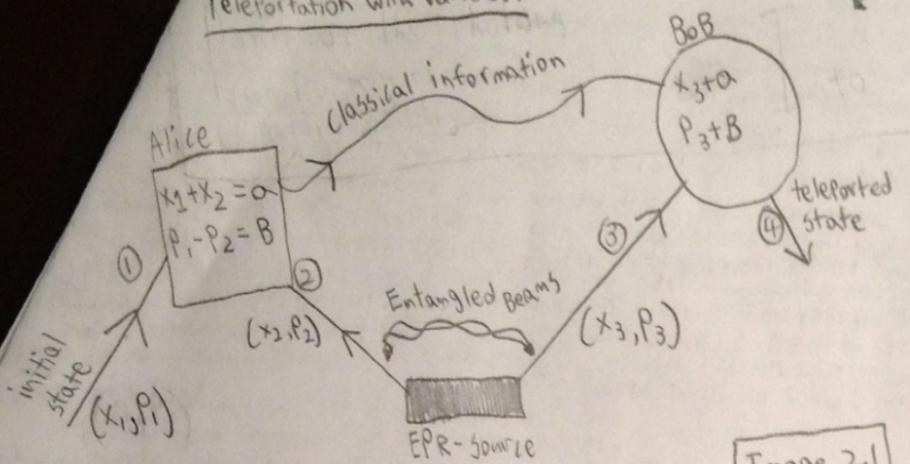
- Any kind of particle(s) can be entangled with other particles.

**look and Read this Before you Start Reading this Page!**

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**NOTE:**  $\alpha = \alpha$  and  $B = B$ , the  $\alpha$  and  $B$ 's are not normal  $a$  and  $b$ . Don't think of them as the letters

### Teleportation with variables:



This uses Position and momentum entanglement instead of Polarisation entanglement. The difference between position and momentum, compared to Polarisation is their representation in terms of certain basis states. PM both need infinite numbers of basis states. Position eigenstates and momentum eigenstates form an infinite-dimensional Hilbert space. The Polarisation of a particle however expresses as the superposition of only two basis states. (Polarisation has two-dimensional Hilbert space).

Similar to Image 1 (100%).

Let's say Alice has a quantum particle with a certain position of  $x_1$  and momentum  $p_1$ , and she wishes to send the quantum info to Bob. Due to Heisenberg uncertainty between  $x$  and  $p$ , which follows the fact that operators for position and momentum do not commute  $[x, \hat{p}] = i\hbar$ , Alice cannot measure both  $x_1$  and  $p_1$  with arbitrary precision. Therefore quantum mechanics forbids Alice to get the info she wishes to transfer. The way out of this dilemma is conceptually the same as the protocol. A EPR source has to be distributed between Alice and Bob. However the two particles in the EPR-source is not entangled in their position and momentum. Let's say the entanglement of particles 2 and 3 is described as:

$$x_2 + x_3 = 0, \text{ and } p_2 - p_3 = 0 \quad (3.45)$$

The properties of the individual particles,  $x_1, x_2, p_1$ , and  $p_2$  are completely undetermined by (3.45). Instead their joint properties are defined. Note that the operators  $\hat{x}$  and  $\hat{p}$  do not commute for each particle, but the operators for  $(x_2 + x_3)$  and  $(p_2 - p_3)$  do commute as a result of relative minus sign between the addition of positions and the addition of momenta. Therefore the entangled states  $(x_2 + x_3)$  and  $(p_2 - p_3)$ , can be measured with an arbitrary accuracy.

Alice then would have to do a Bell-state measurement on particle 1 and 2. That is, the state of particle 1 and 2 is projected onto an entangled state. There will be 4 possible outcomes for a particle in a internal (polarisation) state, for the Bell-state measurement. In the present case the measurement by Alice yields:

$$x_1 + x_2 = a, \text{ and } p_1 - p_2 = b$$

where  $a$  and  $b$  are two real numbers which both have a continuous range of possible values. This indicates that the measurement of the sum of positions and the difference in momenta of those two particles requires projection onto an  $(\infty)$ -dim Hilbert space.

So as a result of the initial entanglement ( $x_2 + x_3 = 0$  and  $p_2 - p_3 = 0$ ) and Alice's measurement ( $x_1 + x_2 = a$  and  $p_1 - p_2 = b$ ), the quantum state that Bob has is:

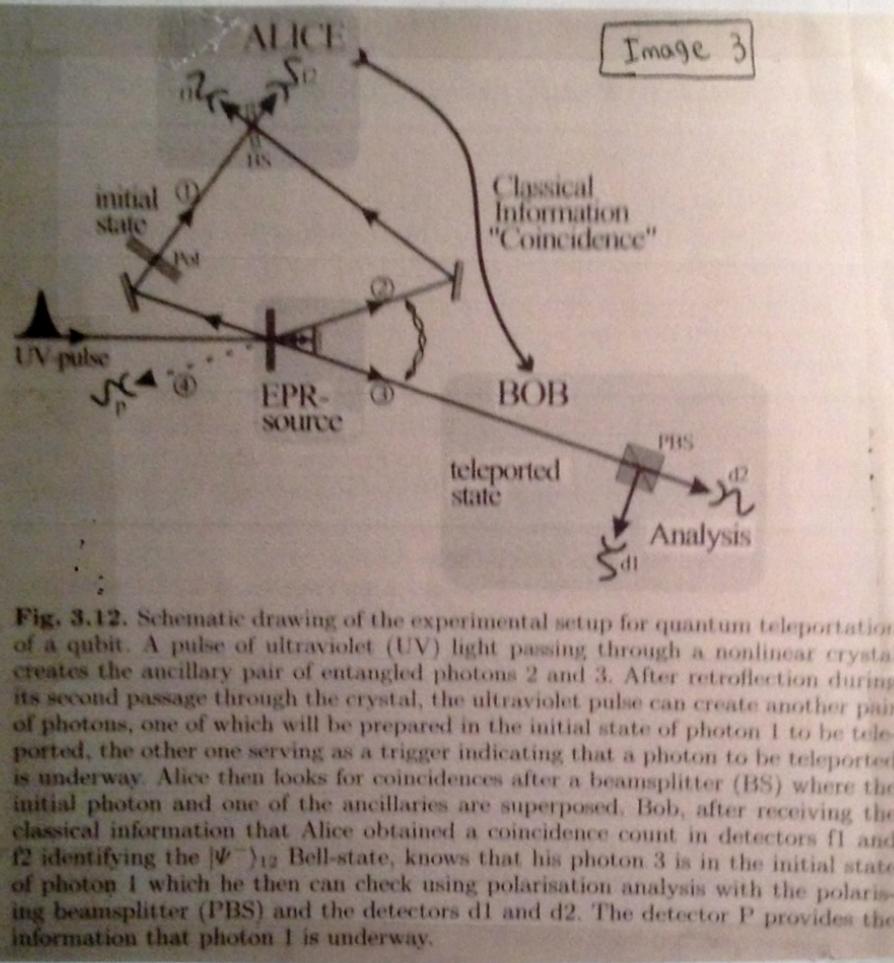
$$x_3 = x_1 - a, \text{ and } p_3 = p_1 - b$$

Now to complete the quantum teleportation protocol, Alice sends her measurements, values of  $a$  and  $b$ , and then Bob will displace the position and momentum of his particle by  $a$  and  $b$ , respectively. The final result is Bob has his particle 3 in the initial quantum state of particle 1.

## Quantum Dense Coding and Quantum teleportation

To avoid photons 1 and 2 from being distinguished by their arrival times at the detector, which would eliminate the possibility of performing the Bell-state measurement, the following technique is used. Photon 2, which is entangled to photon 3, is pulsed parametric down-conversion. The pump pulse, made from a frequency doubled mode-locked titanium-sapphire laser, is 200 fs long. The pulse is then reflected back through the crystal to create a second pair of photons 1 and 4. Photon 4 is used as a trigger to indicate the presence of photon 1. Photons 1 and 2 are now located with in 200 fs long pulses, which we can turn a variable delay so that maximal spatial overlap of the photons at the detectors is obtained. However, this does not yet guarantee indistinguishability upon detection since the entangled down-converted photons typically have a coherence length corresponding a 50 fs long wave packet, which is shorter from pump lasers' pulses. Therefore coincident detection of photons 1 and 2 with their partners 3 and 4 with a time resolution better than 50 fs could identify which photons were created together. To achieve indistinguishability on the detection, the photon wave packets should be stretched at length substantially longer than the pump pulse. In this experiment narrow interference filters were placed in front of the detector. These filters cut photon wave packets with a time duration of the order of 500 fs, which yields a maximum indistinguishability of photons 1 and 2 of about 85%.

Now this brings us to the question of how to prove experimentally that an unknown quantum state can be teleported the set up (Image 3)? One has to show that teleportation can work for set of known non-orthogonal states. The non-orthogonal state test is important for quantum entanglement and teleportation.



**Fig. 3.12.** Schematic drawing of the experimental setup for quantum teleportation of a qubit. A pulse of ultraviolet (UV) light passing through a nonlinear crystal creates the ancillary pair of entangled photons 2 and 3. After retroreflection during its second passage through the crystal, the ultraviolet pulse can create another pair of photons, one of which will be prepared in the initial state of photon 1 to be teleported, the other one serving as a trigger indicating that a photon to be teleported is underway. Alice then looks for coincidences after a beamsplitter (BS) where the initial photon and one of the ancillaries are superposed. Bob, after receiving the classical information that Alice obtained a coincidence count in detectors f1 and f2 identifying the  $|\Psi^-\rangle_{12}$  Bell-state, knows that his photon 3 is in the initial state of photon 1 which he then can check using polarisation analysis with the polarising beamsplitter (PBS) and the detectors d1 and d2. The detector P provides the information that photon 1 is underway.

### How to make a qubit:

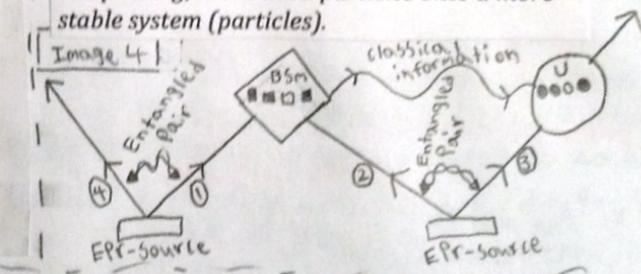
You get a particle, then send microwave-pulse (a very certain one) so you then try to move the particle to a certain position in between spin ( $\uparrow$ ) state and spin ( $\downarrow$ ) state.

Part 1 of 2 for: "Quantum Dense Coding and Quantum teleportation"

### Entangled Swapping:

Look at image 4 to see more in depth how entangled swapping will work. Two EPR sources produce two entangled particle pairs, pair 1-4 and pair 2-3. Two particles, one from each pair (particle 1 and 2) gets a bell-state measurement (BSM). This results in projecting the other two particles 3 and 4 onto an entangled state.

- Teleportation can allow the transfer of fast-deciphering, short-lived particles onto a more stable system (particles).



Book: The Physics of Quantum Information, By: Dirk Bouwmeester, Artur Ekert, and Anton Zeilinger

Pages used for research notes:

7-11, 50-53, 57, 68, 72, 73, and 77-79

Book: Dance of the Photons | From Einstein to Quantum Teleportation, By: Anton Zeilinger

Pages used for research notes:

45-58, 208-217, 224, and 225

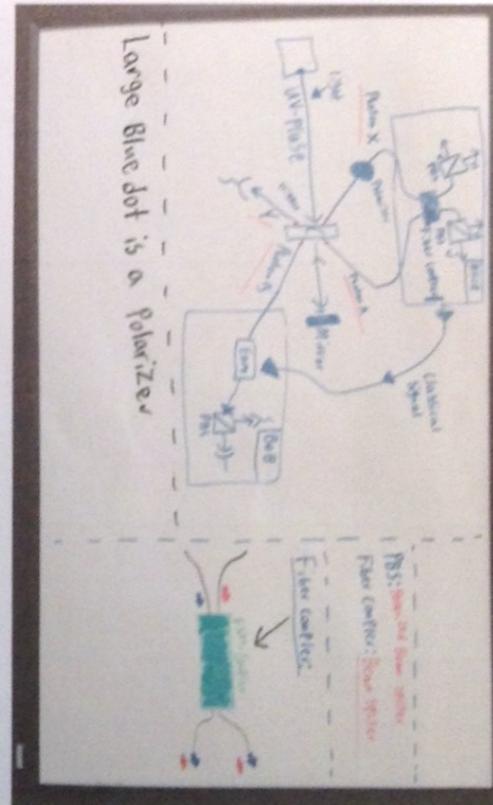
Part 2 of 2 for: "Quantum Dense Coding and Quantum teleportation"

-First a UV-pulse (ultraviolet light) goes through a crystal which makes the photons (photons A and B) entangled, then the pulse hits a mirror and goes through the crystal again making another pair of entangled photons (photons X and Y). Photon A is sent to Alice and photon B is sent to Bob, this pair of entangled photons (A and B) will be used to TP the qubit information. Photon X will go through a polarizer, where it will be imprinted some qubit info. Were photon Y will be used as a trigger to tell if the imprint work or not, also the trigger will tell if the imprint worked when it is no longer entangled with photon X because X is giving new info and Y can no longer recognize X so it stops being entangled with it. When X is no longer entangled with Y it is sent to Alice (NOTE Alice not has photon A and X with her now).

-Second Alice sends photons X and A through a Fiber Coupler (in-fiber beam splitter). This fiber Coupler acts like a 50-50 beam splitter, so for any of the two output, half of the light ends up in one output and half in the other output. Then the two outputs are sent to two polarizing beam splitters (PBS), were the photon(s) takes one path if it is horizontally polarized and the other path if it is vertically polarized (H and V), we use detectors H and V to tell if what position the photons are in. Now we measure these two photons behind the two polarizing beam splitters (PBS), we are then able to project the original photons A and X into a certain entangled state. After that we send the information Alice (the states of photon X and A) has got from the H and V to Bob (NOTE particle X is destroyed now).

-Third Bob gets his photon B, which is entangled with photon A. The classical channel, which is basally how Alice and Bob communicate with each other, tells Bob, which entangled state, was measured by Alice (Alice's measurements). Then what Bob has to do is rotate his photon (manipulate), this can be done by using an electro-optical modulator (EOM). NOTE an EOM work like this: If no voltage is applied the photon just passes through without modification. If the right amount of voltage is applied the photon is rotated in the desired way (modification). We can identify the state of the photon by measuring its polarization, using the polarizing beam splitter (PBS). The PBS can be rotated around the beam axis in order to identify any linear polarization. To know if the teleportation worked or not is if the only correct one of the two detectors behind the beam splitter registers the photon (X) and never the other one. It must be the detector that corresponds to the initial polarization of the teleported photon X.

-Fourth, if all of this was done right, then the teleportation of the qubit was successful and Bob now has photon X with the qubit information.



### Entanglement:

If particle 1 is entangled with particle 2 the following will happen: if you learned particle 1 was at an up beam, then particle 2 will be at a down beam. Then when we make particle 2 move to an up beam, particle 1 will move to a down. Quantum Mechanically this is a two-particle superposition state of the form:

$$\frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + e^{i\chi}|1\rangle_1|0\rangle_2) \quad (1.13)$$

The phase  $\chi$  is just determined by the internal properties of the source and we would assume for  $\chi=0$ . The equation above is called an entangled state. Now the interesting property is that that neither of the two qubits carries a defined value, but as soon as one of the two qubits is subject to a measurement, the result of the measurement being completely random, the other particle will then immediately be found to carry the opposite value. Also distance does not affect the way these entangled particles commute with each other. This is 100% non-logical. The essence of such a violation is that there is no possibility to explain the correlations (relationships) between the two sides (two particles) on the basis of local properties of the qubits alone (also think of it like Schrödinger's cat theory). The quantum correlations between the two sides cannot be understood by assuming that the specific detector on one given side, which registers the particle, is not influenced by the parameter setting, that is, by the choice of the phase for the other particle. (NOTE there are many ways to express the meaning of Bell's inequalities and they're many formal presentations).

Lets now say we entangled three particles instead of two. We will assume a source emits these three particles, as shown at image 3.1. Here is the form of the three entangled particles:

$$\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|1\rangle_2|1\rangle_3) \quad (1.14)$$

This quantum state has many peculiar properties. Also take note that these three particle's qubits carry any information on their own, none of them have a defined value. But as soon as one of the three particles is measured, the other two will assume a well-defined value as long as the measurement is performed in the chosen 0-1 basis. This conclusion holds independent of the spatial separation between the three measurements.

### Entangled and quantum indistinguishability:

A super position means that there is no way to tell which of the two possibilities forming the superposition actually pertains. This is a large part of quantum entanglement. For example, in the state:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2) \quad (1.15)$$

There is no way to tell whether qubit 1 carries the value "0" or "1", and likewise with qubit 2. Yet, if one of the qubits is measured the other one immediately assumes a well-defined quantum state. This can give a person an idea of how entanglement can and will be a LARGE part in teleportation.

There are many ways to produce entangled quantum states. Firstly, one can create a source, which through its physical construction, is such that the quantum states emerging already have the indistinguishability feature. This is realized, for example, by the decay of a spin-0 particle into two spins-1/2 particles under conservation of the internal angular momentum. In that case the two spins of the particles would have to be opposite, and, if no further mechanisms exist which permit one to distinguish the possibilities right at the source, the emerging quantum state is:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2) \quad (1.16)$$

Where  $|\uparrow\rangle$  means particle 1 is spinning up. This state (1.16) has the chance to be rotationally invariant; the two spins are anti-parallel along whichever direction we choose to measure.

A second possibility is that a source might actually produce quantum states of the form of the individual components in the superposition of (1.15), but the states might still be distinguishable in some way. This can happen when in type-II parametric down conversion, where along a certain chosen direction the two emerging photon states are:

$$|H\rangle, |V\rangle_2 \text{ and } |V\rangle, |H\rangle_2 \quad (1.17)$$

This means that either photon 1 is horizontally polarized and photon 2 is vertically polarized. But because different speeds of light for H and V polarized photons inside the down-conversion crystal, the time correlation between the two photons is different in the two cases. Therefore, the two terms in (1.17) can be distinguished by a time measurement and no entangled states results because of this potential to distinguish the two cases. However, in this case too one can still produce entanglement by shifting the two photon-wave packets after their production relative to each other such that they become indistinguishable on the basis of their positions in time. This means the application of a quantum eraser technique where a marker, in this case the relative time ordering, erased such that we obtain quantum indistinguishability resulting in the following state:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + e^{i\chi}|V\rangle_1|H\rangle_2) \quad (1.18)$$

Which is entangled.

The third means of producing entangled states is to project a non-entangled state onto an entangled one. We remark, for example, that an entangled state is never orthogonal to any of its components. Specifically, consider a source producing the non-entangled state:

$$|0\rangle, |1\rangle_2 \quad (1.19)$$

Suppose this state is sent through a filter described by the projection operator:

$$P = |\Psi\rangle_{12}\langle\Psi|_{12} \quad (1.20)$$

Where  $|\Psi\rangle_{12}$  is the state of (1.15). Then the entangled state result would be:

$$\frac{1}{2}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)(\langle 0|_1\langle 1|_2 + \langle 1|_1\langle 0|_2)|0\rangle_1|1\rangle_2 = \frac{1}{2}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2) \quad (1.21)$$

It is no longer normalized to unity because the projection procedure implies a loss of qubits. These three methods stated above are used to produce outgoing entangled states.

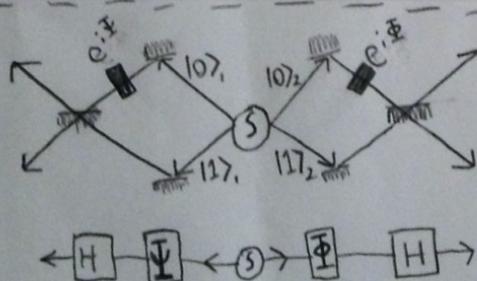


Image 3: A source emits two qubits in an entangled state; Top: A two-particle interferometer verification. Bottom: The principle in terms of one-photon gates.

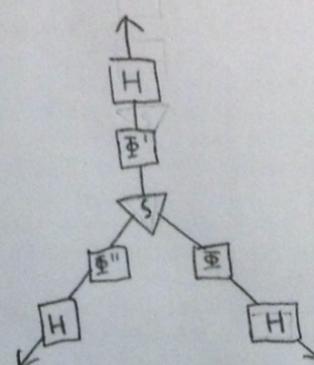


Image 3.1: Three-Particle Entanglement in a so-called GHZ state. Here we show only the representation in terms of our elementary gates.

- Consider the physical realisation in a three-particle interferometer.

Related to Entanglement

# Quantum teleportation Vocab:

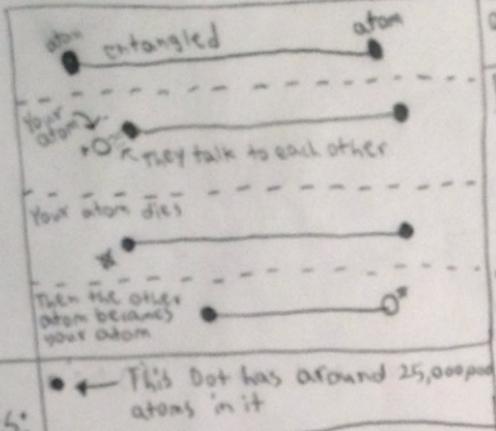
- Spin polarization: A degree to which the spin of a particle's momentum is aligned to a certain direction.
  - Momentum: Something on the move that will take effort to stop.
  - Position: Places where some things is located.
  - Basis (linear algebra): A set of linearly independent vectors that, in a linear combination, can represent every vector in a given vector space, or free module, or, more simply put, which define a "coordinate system" (as long as the basis is given a definite order).
  - Linear independence: There are two different linear independencies: the linear independence of a family of vectors, and the linear independence of a set of vectors.
  - Vector: Any quantity with both a magnitude and a direction. For example, velocity is a vector because it describes both how fast something is moving and in what direction it is moving.
  - Vector space: A mathematical structure formed by a collection of elements called vectors, which can be added together and multiplied by numbers called scalars (real number) in the context. (Note scalars can also be complex, rational numbers, or generally any field.)
  - Real numbers: A value that represents a quantity along a continuous line.
- Rational numbers consist of Integers, whole numbers, Natural numbers, 1, 2, 3, 4, ... and Irrational numbers consist of  $\pi, \sqrt{2}, \sqrt{3}, \dots$
- Mathematical structure: A set or more generally a type, consists of additional mathematical object that, in some manner, attach (or relates) to the set, making it easier to visualize or work with, or endowing the collection with meaning or significance.
  - Set: A collection of distinct objects, considered as an object in its own right. (A collection of elements)  
Ex:  $A = \{1, 2, 3, 4\}$  is a set
  - Elements: The objects that make up a set.
  - Linear combination: An expression constructed from a set of terms by multiplying each term by multiplying each term by a constant and adding the results. (Linear combinations of  $x$  and  $y$  would be expressed by the form  $ax+by$ , where  $a$  and  $b$  are constants.)
  - Amplitude: The maximum displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position. (Is equal to 1/2 of its oscillation path).
  - Equilibrium position: A condition in which all acting influences are canceled by others, resulting in a stable, balanced, or unchanging system.
  - Unitary transformation: A transformation that preserves the inner product: the inner product of two vectors before the transformation is equal to their inner product after the transformation.
  - Inner Product: A vector space with an additional structure called an inner product.
  - Structure: A metric space that is linear and complete and usually infinite-dimensional.
  - Metric Space: A set for which distances between all members (elements) of the set are defined.
  - Orthogonality: Relation of two lines at right angles to one another, and the generalization of this relation into  $n$  dimensions and to a variety of mathematical relations thought of as describing non-overlapping, uncorrelated, or independent objects of some kind.  
"The line segments AB and CD are orthogonal to each other"
  - Amplitude: The maximum displacement or distance by a point on a vibrating body or wave measured from its equilibrium position.
  - Superposition: The theory that a quantum object can be at multiple states, until it is measured.

What is a Bit?

A bit is the basic unit of information in computing and digital communication. A bit has two values: 1 and 0. These values usually mean (true, false), (yes, no), (+, -), etc. Bits are part of binary code. Binary code by the way is text using the binary number system's two digits; 1 and 0.

Ex: A 16-bit word has  $7,000,000,000,000,000,000,000,000$  atoms! This is how computers would spell "Wikipedia".

$W = 01010111$   
 $i = 01101001$   
 $k = 01101011$   
 $j = 01101001$   
 $p = 01110000$   
 $e = 01100101$   
 $d = 01100100$   
 $i = 01101001$   
 $a = 01100001$



### Qubit States:

A Qubit is written a linear combination of  $|0\rangle$  and  $|1\rangle$ :

$$|G\rangle = a|0\rangle + b|1\rangle$$

where  $a$  and  $b$  are probability amplitudes and can in general both be complex numbers

If we were to measure this Qubit, then the probability outcome of  $|0\rangle$  is  $|a|^2$  and  $|1\rangle$  is  $|b|^2$ . Because the absolute squares of the amplitudes equate to probabilities, this follows that  $a$  and  $b$  must be constrained by the equation:

$$|a|^2 + |b|^2 = 1$$

Simply because this ensures you must measure either one or the other.

A Bell state is defined as a maximally entangled quantum state of two qubits. A qubit held by Alice can be 1 or 0, but if she measures the Bell state the outcome would be random. Also it would have a probability of 1/2. But if Bob then measures his qubit, the outcome would be the same as Alice's. If Bob measured, he would also get a random outcome, but if Alice and Bob communicated they would find out that the outcomes are random, but they are correlated.

### What is a Qubit?

A qubit is a unit of quantum information—the quantum analogue of the classical bit.

### Bit vs Qubit:

A bit can value a 1 or 0, well a qubit can value a 1, 0, and both 1 and 0 at the same time. [Shor's code:  $|0\rangle = |0\rangle, |1\rangle = |1\rangle, |2\rangle = \sqrt{|0\rangle + |1\rangle}, |3\rangle = \sqrt{|0\rangle - |1\rangle}$ ] Same How much numbers a bit and a qubit can hold, ex:

- 8 bit = 1 number between 0 and 255
- 8 qubit = All 256 numbers at once
- 10 qubits = 1024 numbers
- 11 qubits = 2048 numbers
- 100 qubits = 3,257,650,600,228,229,401,496,703,205,376 numbers

### Vocab:

• Linear combination: A expression made of a set of terms by multiplying each term by an constant and adding the results. This can be written as  $a_1x_1 + a_2x_2 + \dots + a_nx_n$ , where  $a_i$  and  $x_i$  are constants

• Probability Amplitude: A complex number used in describing the behaviour of systems.

ex:

• Complex Numbers: A number expressed in  $a+bi$ , where  $a$  and  $b$  are real numbers and  $i$  is an imaginary number.

• Imaginary number ( $i$ ): When  $i$  is squared, it gives a negative result.

• Correlate: Having a mutual relationship or connection; in which one thing affects or depends on another.



A large portion of a single atom can be entangled with another atom, and the whole atom can be entangled with a classical bit. Similarly to a classical bit where the state of a transistor is a processor, the interpretation of a qubit is a hard disk and the presence of certain bits in a table can all be used to represent data in the same computer, an eventual quantum computer is likely to use random combinations of qubits in its storage.

There are various proposals. Several physical implementations which approximate non-local mathematics in certain respects were experimentally realized. Similarly to a classical bit where the state of a transistor is a processor, the interpretation of a qubit is a hard disk and the presence of certain bits in a table can all be used to represent data in the same computer, an eventual quantum computer is likely to use random combinations of qubits in its storage.

The following is an incomplete list of physical implementations of qubits, and the choices of basis are by convention:

Implementation	Number of qubits	State	Implementation	Number of qubits	State	Implementation	Number of qubits	State
Electromagnetic	1	$ 0\rangle$	Electromagnetic	1	$ 0\rangle$	Electromagnetic	1	$ 0\rangle$
Electromagnetic	2	$ 00\rangle$	Electromagnetic	2	$ 00\rangle$	Electromagnetic	2	$ 00\rangle$
Electromagnetic	3	$ 000\rangle$	Electromagnetic	3	$ 000\rangle$	Electromagnetic	3	$ 000\rangle$
Electromagnetic	4	$ 0000\rangle$	Electromagnetic	4	$ 0000\rangle$	Electromagnetic	4	$ 0000\rangle$
Electromagnetic	5	$ 00000\rangle$	Electromagnetic	5	$ 00000\rangle$	Electromagnetic	5	$ 00000\rangle$
Electromagnetic	6	$ 000000\rangle$	Electromagnetic	6	$ 000000\rangle$	Electromagnetic	6	$ 000000\rangle$
Electromagnetic	7	$ 0000000\rangle$	Electromagnetic	7	$ 0000000\rangle$	Electromagnetic	7	$ 0000000\rangle$
Electromagnetic	8	$ 00000000\rangle$	Electromagnetic	8	$ 00000000\rangle$	Electromagnetic	8	$ 00000000\rangle$
Electromagnetic	9	$ 000000000\rangle$	Electromagnetic	9	$ 000000000\rangle$	Electromagnetic	9	$ 000000000\rangle$
Electromagnetic	10	$ 0000000000\rangle$	Electromagnetic	10	$ 0000000000\rangle$	Electromagnetic	10	$ 0000000000\rangle$
Electromagnetic	11	$ 00000000000\rangle$	Electromagnetic	11	$ 00000000000\rangle$	Electromagnetic	11	$ 00000000000\rangle$
Electromagnetic	12	$ 000000000000\rangle$	Electromagnetic	12	$ 000000000000\rangle$	Electromagnetic	12	$ 000000000000\rangle$
Electromagnetic	13	$ 0000000000000\rangle$	Electromagnetic	13	$ 0000000000000\rangle$	Electromagnetic	13	$ 0000000000000\rangle$
Electromagnetic	14	$ 00000000000000\rangle$	Electromagnetic	14	$ 00000000000000\rangle$	Electromagnetic	14	$ 00000000000000\rangle$
Electromagnetic	15	$ 000000000000000\rangle$	Electromagnetic	15	$ 000000000000000\rangle$	Electromagnetic	15	$ 000000000000000\rangle$
Electromagnetic	16	$ 0000000000000000\rangle$	Electromagnetic	16	$ 0000000000000000\rangle$	Electromagnetic	16	$ 0000000000000000\rangle$
Electromagnetic	17	$ 00000000000000000\rangle$	Electromagnetic	17	$ 00000000000000000\rangle$	Electromagnetic	17	$ 00000000000000000\rangle$
Electromagnetic	18	$ 000000000000000000\rangle$	Electromagnetic	18	$ 000000000000000000\rangle$	Electromagnetic	18	$ 000000000000000000\rangle$
Electromagnetic	19	$ 0000000000000000000\rangle$	Electromagnetic	19	$ 0000000000000000000\rangle$	Electromagnetic	19	$ 0000000000000000000\rangle$
Electromagnetic	20	$ 00000000000000000000\rangle$	Electromagnetic	20	$ 00000000000000000000\rangle$	Electromagnetic	20	$ 00000000000000000000\rangle$
Electromagnetic	21	$ 000000000000000000000\rangle$	Electromagnetic	21	$ 000000000000000000000\rangle$	Electromagnetic	21	$ 000000000000000000000\rangle$
Electromagnetic	22	$ 0000000000000000000000\rangle$	Electromagnetic	22	$ 0000000000000000000000\rangle$	Electromagnetic	22	$ 0000000000000000000000\rangle$
Electromagnetic	23	$ 00000000000000000000000\rangle$	Electromagnetic	23	$ 00000000000000000000000\rangle$	Electromagnetic	23	$ 00000000000000000000000\rangle$
Electromagnetic	24	$ 000000000000000000000000\rangle$	Electromagnetic	24	$ 000000000000000000000000\rangle$	Electromagnetic	24	$ 000000000000000000000000\rangle$
Electromagnetic	25	$ 0000000000000000000000000\rangle$	Electromagnetic	25	$ 0000000000000000000000000\rangle$	Electromagnetic	25	$ 0000000000000000000000000\rangle$
Electromagnetic	26	$ 00000000000000000000000000\rangle$	Electromagnetic	26	$ 00000000000000000000000000\rangle$	Electromagnetic	26	$ 00000000000000000000000000\rangle$
Electromagnetic	27	$ 000000000000000000000000000\rangle$	Electromagnetic	27	$ 000000000000000000000000000\rangle$	Electromagnetic	27	$ 000000000000000000000000000\rangle$
Electromagnetic	28	$ 0000000000000000000000000000\rangle$	Electromagnetic	28	$ 0000000000000000000000000000\rangle$	Electromagnetic	28	$ 0000000000000000000000000000\rangle$
Electromagnetic	29	$ 00000000000000000000000000000\rangle$	Electromagnetic	29	$ 00000000000000000000000000000\rangle$	Electromagnetic	29	$ 00000000000000000000000000000\rangle$
Electromagnetic	30	$ 000000000000000000000000000000\rangle$	Electromagnetic	30	$ 000000000000000000000000000000\rangle$	Electromagnetic	30	$ 000000000000000000000000000000\rangle$
Electromagnetic	31	$ 0000000000000000000000000000000\rangle$	Electromagnetic	31	$ 0000000000000000000000000000000\rangle$	Electromagnetic	31	$ 0000000000000000000000000000000\rangle$
Electromagnetic	32	$ 00000000000000000000000000000000\rangle$	Electromagnetic	32	$ 00000000000000000000000000000000\rangle$	Electromagnetic	32	$ 00000000000000000000000000000000\rangle$
Electromagnetic	33	$ 000000000000000000000000000000000\rangle$	Electromagnetic	33	$ 000000000000000000000000000000000\rangle$	Electromagnetic	33	$ 000000000000000000000000000000000\rangle$
Electromagnetic	34	$ 0000000000000000000000000000000000\rangle$	Electromagnetic	34	$ 0000000000000000000000000000000000\rangle$	Electromagnetic	34	$ 0000000000000000000000000000000000\rangle$
Electromagnetic	35	$ 00000000000000000000000000000000000\rangle$	Electromagnetic	35	$ 00000000000000000000000000000000000\rangle$	Electromagnetic	35	$ 00000000000000000000000000000000000\rangle$
Electromagnetic	36	$ 000000000000000000000000000000000000\rangle$	Electromagnetic	36	$ 000000000000000000000000000000000000\rangle$	Electromagnetic	36	$ 000000000000000000000000000000000000\rangle$
Electromagnetic	37	$ 0000000000000000000000000000000000000\rangle$	Electromagnetic	37	$ 0000000000000000000000000000000000000\rangle$	Electromagnetic	37	$ 0000000000000000000000000000000000000\rangle$
Electromagnetic	38	$ 00000000000000000000000000000000000000\rangle$	Electromagnetic	38	$ 00000000000000000000000000000000000000\rangle$	Electromagnetic	38	$ 00000000000000000000000000000000000000\rangle$
Electromagnetic	39	$ 000000000000000000000000000000000000000\rangle$	Electromagnetic	39	$ 000000000000000000000000000000000000000\rangle$	Electromagnetic	39	$ 000000000000000000000000000000000000000\rangle$
Electromagnetic	40	$ 00\rangle$	Electromagnetic	40	$ 00\rangle$	Electromagnetic	40	$ 00\rangle$
Electromagnetic	41	$ 000\rangle$	Electromagnetic	41	$ 000\rangle$	Electromagnetic	41	$ 000\rangle$
Electromagnetic	42	$ 00\rangle$	Electromagnetic	42	$ 00\rangle$	Electromagnetic	42	$ 00\rangle$
Electromagnetic	43	$ 000\rangle$	Electromagnetic	43	$ 000\rangle$	Electromagnetic	43	$ 000\rangle$
Electromagnetic	44	$ 00\rangle$	Electromagnetic	44	$ 00\rangle$	Electromagnetic	44	$ 00\rangle$
Electromagnetic	45	$ 000\rangle$	Electromagnetic	45	$ 000\rangle$	Electromagnetic	45	$ 000\rangle$
Electromagnetic	46	$ 00\rangle$	Electromagnetic	46	$ 00\rangle$	Electromagnetic	46	$ 00\rangle$
Electromagnetic	47	$ 000\rangle$	Electromagnetic	47	$ 000\rangle$	Electromagnetic	47	$ 000\rangle$
Electromagnetic	48	$ 00\rangle$	Electromagnetic	48	$ 00\rangle$	Electromagnetic	48	$ 00\rangle$
Electromagnetic	49	$ 000\rangle$	Electromagnetic	49	$ 000\rangle$	Electromagnetic	49	$ 000\rangle$
Electromagnetic	50	$ 00\rangle$	Electromagnetic	50	$ 00\rangle$	Electromagnetic	50	$ 00\rangle$
Electromagnetic	51	$ 000\rangle$	Electromagnetic	51	$ 000\rangle$	Electromagnetic	51	$ 000\rangle$
Electromagnetic	52	$ 00\rangle$	Electromagnetic	52	$ 00\rangle$	Electromagnetic	52	$ 00\rangle$
Electromagnetic	53	$ 000\rangle$	Electromagnetic	53	$ 000\rangle$	Electromagnetic	53	$ 000\rangle$
Electromagnetic	54	$ 00\rangle$	Electromagnetic	54	<			

Photon with quantum state  $|\phi\rangle$

EPR pair (quantum channel)

A

Two bits (classical channel)

B

Photon with quantum state  $|\phi\rangle$

time

- 1) An EPR pair is generated, one qubit sent to location A, the other to B.
- 2) At location A, a Bell measurement of the EPR pair qubit and the qubit to be teleported (the quantum state  $|\phi\rangle$ ) is performed, yielding one of four possibilities, which can be encoded in two classical bits of information. Both qubits at location A are then discarded.
- 3) Using the classical channel, the two bits are sent from A to B. (This is the only potentially time-consuming step, due to speed-of-light considerations.)
- 4) As a result of the measurement performed at location A, the EPR pair qubit at location B is in one of four possible states. Of these four possible states, one is identical to the original quantum state  $|\phi\rangle$ , and the other three are closely related. Which of these four possibilities it actually is, is encoded in the two classical bits. Knowing this, the qubit at location B is modified in one of three ways, or not at all, to result in a qubit identical to  $|\phi\rangle$ , the qubit that was chosen for teleportation.

other:

